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GENERATION OF GRACEFUL TREES

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ABSTRACT

In this paper, a new family of trees is generated by combining the given trees, and it proves that the result tree T_1 is graceful and admits α - valuation. Further, by recursively attaching a given tree with the resultant, generates a bigger family of trees T_i for$i\ \geq 2$, and we prove that such trees are graceful and have α - valuation.

KEYWORDS: Trees, Recursive Attachment, Graceful Labeling, Graceful Trees, α - Valuation

1. INTRODUCTION

Let G (V,E) be a simple connected graph. A function φ is called a graceful labeling of a graph G with m edges, if φ is an injection from the vertices of G to the set $\{0,1,2,\ldots,m\}$ such that when each edge uv is assigned the label$|\varphi(u)-\ \varphi(v)|$, the resulting edge labels are distinct. A graceful labelling φ is called an α (valuation) of G, if there exists an integer λ such that for each edge $e=\ uv$, either $\varphi(u)\leq\lambda<\ \varphi(v)$ or $\varphi(v)\leq\lambda<\ \varphi(u)$.

The Ringel – Kotzig conjecture, all trees are gracefully is a long standing conjecture. Special classes of trees have been verified, but still this conjecture remains hard to prove. Also, many authors have given different methods to yield bigger graceful trees from the known, graceful tree [1], [2], [4] and exhaustively provided in the dynamic survey by Gallian [3]. In this direction, using the method of combining trees as defined by Sethuraman [6], a new family of trees have been constructed recursively from the given tree and further it is proved that the resultant tree is graceful and admits α - valuation.

Let G be any connected graph having p admissible vertices (a vertex of degree at least two). Let H be any graph and take p copies of H. Thus, $G\oplus H$ is the graph obtained by merging a chosen vertex of each copy of H with every admissible ververtex G. Note that the method of combining trees follows from [6] and in that paper, the choice of the graphs G and H are caterpillars, but here the graph H is a tree and its construction follows.

Let us define the following terms that are used in this paper that are followed in [6]. If T is a caterpillar, then

a) A vertex v of T which has at most one vertex of degree greater than or equal to two is called as penultimate vertex. Denote one of its penultimate vertices as head and the other as tail.

b) The graph obtained by subdividing the unique edge incident with the head of T is represented as T^{-} and hence T^{-} is also a caterpillar.

c) The unique edge incident with the head, which has the other end having degree at least two is called the neck edge.

Let T' and T'' be any two copies of the caterpillar T. Then, S is the graph obtained by merging the head of T', T'' and T^{-} together. See Figures 1, 2 and 3.

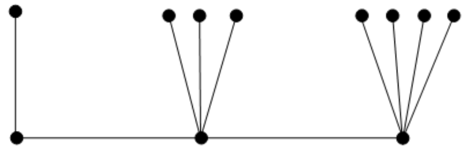

Figure 1: A Caterpillar *T*

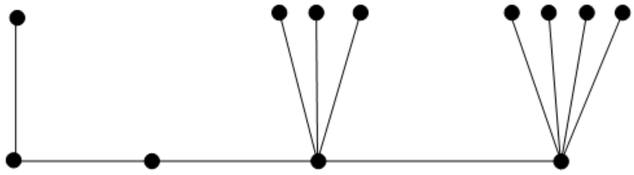

Figure 2: The Caterpillar T^{-}

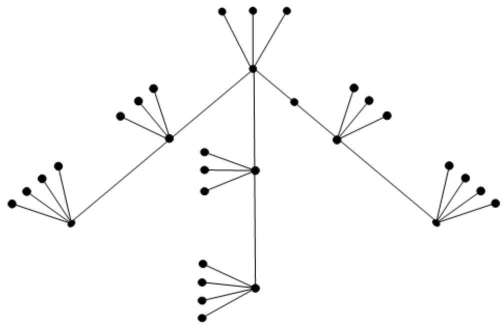

Figure 3: The Tree S (Combining the Head of T', T'' and T^{-} Together)

Let T_0 be any caterpillar and $S_1 \in S$. Now, let us define $T_1 = T_0 \oplus S_1$, by attaching the head of each copy S_1 With all the admissible vertices of T_0.

In the similar way, for $i \geq 2$, define $T_i = T_{i-1} \oplus S_i$ in a recursive manner and in this paper, we prove that T_i is graceful and admits α - valuation.

2. T_1 ADMITS α - VALUATION

In this section we prove that the tree T_1, as constructed above is graceful and further admits α - valuation.

Theorem 2.1

The tree T_1 *admits* α *- valuation.*

Proof

Let m be the number of edges of the tree T_1.

Now label the admissible vertices of T_1 from tail to head and head to tail of each copy of S_1^i of S_1 and let it be $u_1, u_2, u_3, \ldots u_t$.

Introduce artificial edges between some pair of vertices of S_1^i in such a way that, by joining the tail of T'' and T^{-} by an arc and also join the neck vertex of T^{-} of i^{th} copy to $(i+1)^{th}$ copy, if i is odd. Otherwise, join the tail of T'' and T^{-} by an arc and also join the tails of T' between i^{th} copy to $(i+1)^{th}$ copy. Thus T_1 is the tree along with the artificial edges (dotted lines). Refer Figure. 5.

Represent the obtained tree with the labelled admissible vertices as a bipartite graph with the bipartition (A, B) as follows,

$A = \{u_{2i-1} : 1 \leq i \leq t\}$ and $B = \{u_{2i} : 1 \leq i \leq t\}$.

Clearly (A, B) defines the bipartition of T_1. Refer Figure. 6.

Now label the vertices of A, that are, $a_1, a_2, a_3, \dots, a_p$ by $m, m-1, m-2, \dots, m-(p-1)$ and the vertices of B, that is, $b_1, b_2, b_3, \dots, b_q$ by $0, 1, 2, 3, \dots, q-1$. Refer Figure. 7.

Clearly, all the edge values of being distinct and varies from $1, 2, 3, \dots, m$ and by removal of all artificial edges results the grace labelling of T_1 and further admits α - valuation (refer Figure. 8) With $\lambda = m - p$.

Theorem 2.2

For $i \geq 2$, *the tree* T_i *admits* α *- valuation.*

Proof

Consider the tree T_{i-1} and by induction, assume that T_{i-1} has graceful numbering.

First, ignore the grace labelling of T_{i-1} and consider the treeS_{i-1}, then construct the treeT_i, that is, $T_{i-1} \oplus S_{i-1}$ by attaching the head of each copy S_{i-1} with all admissible vertices of T_{i-1}. Let M be the number of edges of T_i. Then introduce the artificial edges, and in a similar way represent the resultant tree as a bipartite graph (A, B), as defined in theorem 2.1. Label the vertices of A that is, $a_1, a_2, a_3, \dots, a_p$ by $M, M-1, M-2, \dots, M-(p-1)$ and the vertices of B, that is, $b_1, b_2, b_3, \dots, b_q$ by $0, 1, 2, 3, \dots, q-1$. Clearly, all the edge values of a being distinct and varies from $1, 2, 3, \dots M$ and by removal of all artificial edges results the grace labeling of T_1 and further it has α - valuation with $\lambda = M - p$. Refer Figure 9 and Figure 10.

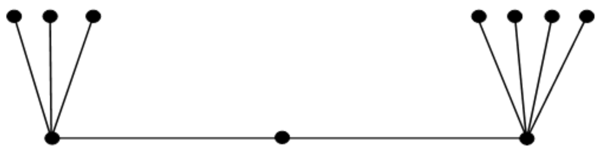

Figure 4: The Caterpillar T_0

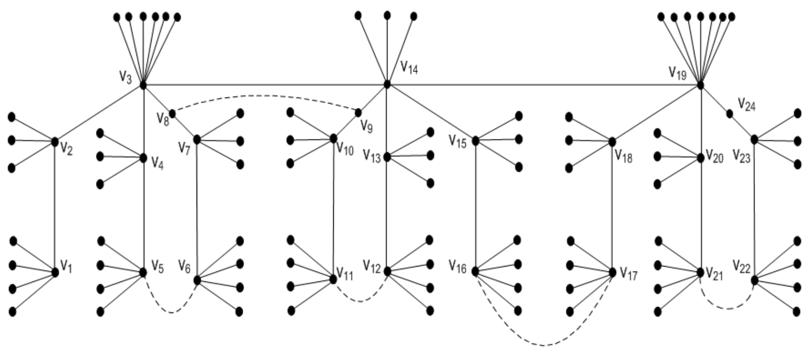

Figure 5: T_1 with the Artificial Edges (T_0 From Figure.4 and S_1As Figure.3)

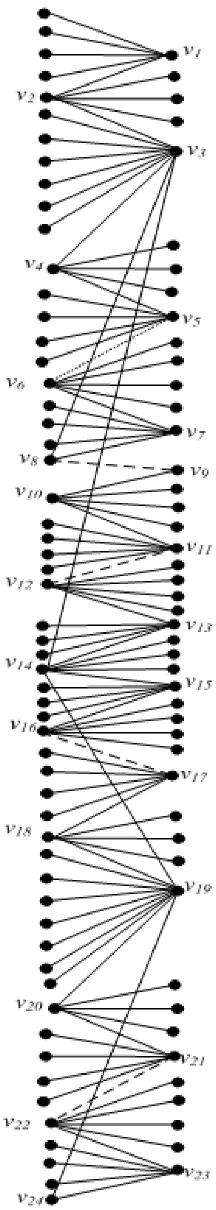

Figure 6: Bipartition of T_1

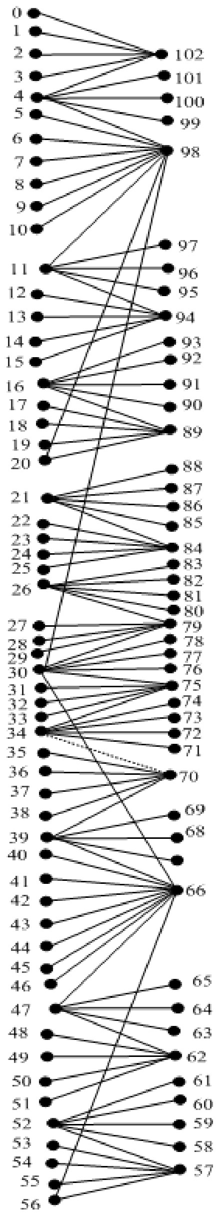

Figure 7: α - Valuation of T_1

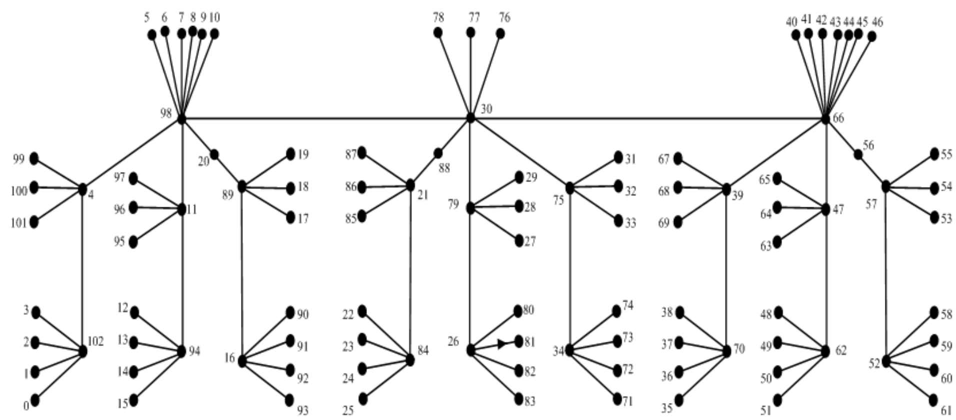

Figure 8: α - Valuation of $T_1 = T_0 \oplus S_1$

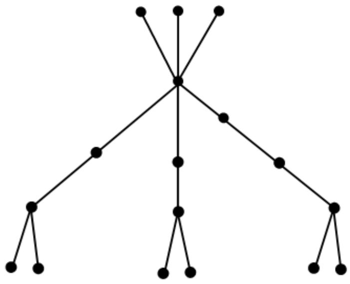

Figure 9: The Caterpillar of S_2

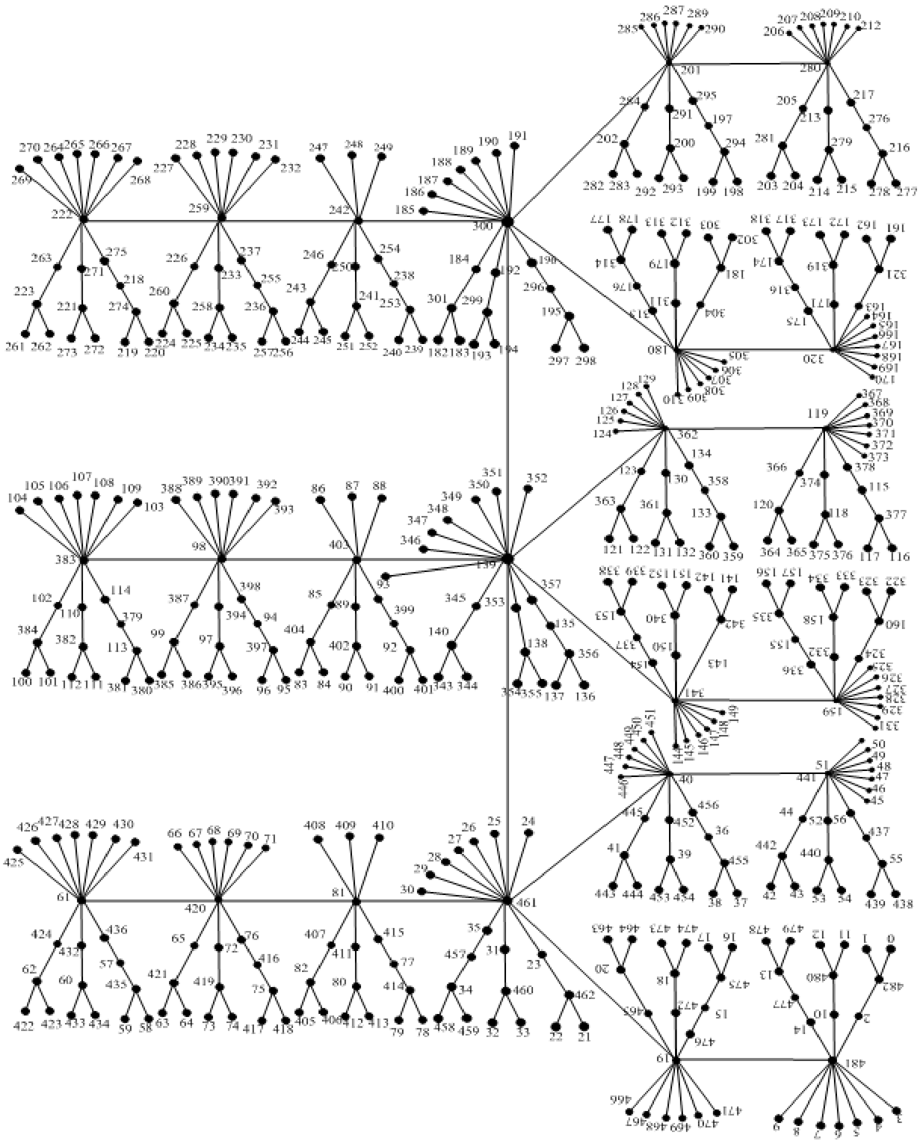

Figure 10: α - Valuation of $T_2 = T_1 \oplus S_2$

CONCLUSIONS

In this paper we have proved that by recursively attaching a given tree with the caterpillar generates a family of graceful trees. Further we conjecture that this recursive attachment still generate a family graceful trees by choosing not necessarily a caterpillar, as the attachment graph (tree).

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